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# CONTROL OF FLOWING LIQUID FILMS BY ELECTROSTATIC FIELDS IN SPACE

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## ABSTRACT

A novel type of lightweight space radiator has been proposed which employs internal electrostatic fields to stop coolant leaks from punctures caused by micrometeorites or space debris. Extensive calculations have indicated the feasibility of leak stoppage without film destabilization for both stationary and rotating designs. Solutions of the evolution equation for a liquid-metal film on an inclined plate, using lubrication theory for low Reynolds numbers, Karman-Pohlhausen quadratic velocity profiles for higher Reynolds numbers, and a direct numerical solution are shown. For verification an earth-based falling-film experiment on a precisely-vertical wall with controllable vacuum on either side of a small puncture is proposed. The pressure difference required to start and to stop the leak, in the presence and absence of a strong electric field, will be measured and compared with calculations. Various parameters, such as field strength, film Reynolds number, contact angle, and hole diameter will be examined. A theoretical analysis will be made of the case where the electrode is close enough to the film surface that the electric field equation and the surface dynamics equations are coupled. Preflight design calculations will be made in order to transfer the modified equipment to a flight experiment.

## INTRODUCTION

Electrostatic forces are inherently weak relative to gravitational forces, and hence have been of limited use in controlling liquid jets, films, etc. In microgravity environments, however, the situation is quite different, and some interesting new applications arise. The usages of electric fields for effecting and improving phasic separations, ionic separations, and molecular separations are well-known. However, the effect of strong electrostatic fields on free-surface flows, and in particular, the flow of thin films on a solid substrate is quite new, and potentially of considerable practical importance in microgravity.

Electric fields tend to pull a conductive (or dielectric) material into a non-conductive (or near vacuum) adjacent region. When applied to a free-flowing film, the electric field produces a surface wave, which may become unstable and contact the electrode above the film which produces the electric field. With proper design, however, the surface wave amplitude is limited to a safe value, and the wave is washed harmlessly away downstream. The tensile stress exerted by the field opposes the hydrostatic and vapor pressure tending to drive the liquid through a puncture caused by a collision with a micrometeorite or space debris. Hence this promises to be an effective device for stopping leaks from relatively large micrometeorite collisions, which

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would make membrane-type pumped-loop space radiators practical. This should result in a considerable weight saving over heat pipe radiators.

The theory and possible designs have been extensively investigated by Kim, et al. (Refs. 1-5). These detailed analytical and design calculations have pointed to the feasibility and stability of several variants, both rotating and non-rotating, of the electrostatic liquid-film radiator concept. To test it experimentally would seem to require a microgravity environment. This is because on earth gravitational forces are many thousands of times greater than electrostatic forces. However, a relatively inexpensive, but sophisticated experiment has been devised which will allow preliminary testing of the leak-stopping concept on earth in our laboratory. Extensive tests will be performed over a three year period, after which a design study will be conducted for flight experiments for further tests under microgravity conditions.

In addition, we wish to examine a theoretical problem which extends the previous analytical work. Basic to the previous work was the assumption that the electric field (Laplace) equation is uncoupled from the evolution equation for the surface waves, since the distance from the electrode to the film is at least an order of magnitude greater than the thickness of the base film. When this is no longer true, the coupled equation system must be solved numerically, presenting a challenging problem for boundary integral methods.

#### THIN FILM FLOW ON AN INCLINED FLAT MEMBRANE

The simplest problem is flow of a thin film on an inclined flat plate, where the film is both stabilized and driven by gravity. In space gravity would be replaced by centrifugal force, produced by rotation and/or azimuthal flow along a curved surface, such as a cylinder. Figure 1 shows a flat membrane inclined at an angle  $\beta$  to the direction of gravity, above which a liquid film flows. Above the film is suspended a charged foil (electrode), whose field causes a standing trough, followed by a crest, in the film surface. The equations are the Laplace equation in the vacuum, and scaled continuity and momentum equations in the liquid. At the free liquid surface the boundary conditions are that the tangential electric field, the normal displacement field, the tangential stress and the normal stress are continuous, together with a kinematic condition on the fluid particles. The electric field appears only in the normal stress boundary condition. There are four dimensionless constants (Reynolds (Re), Froude (Fr), capillary (Ca) and electrostatic field numbers (K)) which need to be specified to solve the problem.

Here the capillary number is

$$Ca = \frac{2\mu U_0}{\sigma} \quad (1)$$

where  $\mu$  is the liquid viscosity,  $\sigma$  is surface tension and  $U_0$  is the average liquid velocity. The electrostatic field number is given by

$$K = \frac{\epsilon_0 d F^2}{16 \pi \mu U_0}, \quad (2)$$

where  $\epsilon_0$  is the vacuum dielectric constant,  $F$  is the electric field strength and  $d$  is the mean film thickness.

Since the length of the electrode in the direction of flow is effectively infinite, or at least large compared to the mean film thickness, a thin-film analysis is appropriate. This was performed for Reynolds numbers of  $O(1)$  (proposed experiment) and of  $O(1/\xi)$  (proposed radiator), where  $\xi \ll 1$  is the ratio of the film thickness to a characteristic length in the flow direction. In the former case lubrication theory can be employed, and a nonlinear evolution equation can be derived, in a similar fashion as many other investigators (Ref. 6), except that an electrostatic term has been added on the right-hand side.

The electric field is determined by solving the Laplace equation for the electric potential  $\phi(x,y)$  in the fluid  $\phi_f$ , and the electric potential,  $\phi_v$ , in the vacuum region above the fluid but below the charged plate (Figure 1). The fluid region,  $V_f$ , is defined by  $0 \leq y \leq h(x,t)$  and  $-\infty < x < \infty$ , where  $y = h(x,t)$  is the height of the film above the inclined plane, and the vacuum region,  $V_v$ , is defined by the strip  $-\infty < x < \infty$  and  $h(x,t) \leq y \leq \hat{H}$ . The boundary conditions are that

$$\phi(x, \hat{H}) = F\hat{H} \phi(x), \quad \text{for } y = \hat{H}, \quad (3a)$$

$$\phi = 0, \quad \text{for } y = 0. \quad (3b)$$

The function  $\phi(x)$  is a given dimensionless function of  $x$ , and the product  $F\hat{H}$  is a constant with units of electric potential. Along  $y = h(x,t)$  we have the boundary conditions that the tangential electric field and the normal displacement field are continuous (Landau, et al., 1984)

$$\phi^f(x, h, t) = \phi^v(x, h, t), \quad \epsilon_f \frac{\partial \phi^f}{\partial n} = \epsilon_0 \frac{\partial \phi^v}{\partial n}. \quad (4a, b)$$

Here  $\epsilon_f$  is the dielectric constant of the fluid and the partial derivative is in the direction of the outward unit normal,  $n$ , to the interface.

Following the now-standard procedures of Benney (Ref. 6) and Gjevik (Ref. 7) one can derive the nonlinear long-wave evolution equation for the film thickness as a function of time and space.

$$\frac{\partial h}{\partial t} + 3h^2 \frac{\partial h}{\partial x} + \xi \frac{\partial}{\partial x} \left[ \frac{6}{5} Re h^6 \frac{\partial h}{\partial x} - \cot(\beta) h^3 \frac{\partial h}{\partial x} + \frac{2}{3} \frac{\xi^2}{Ca} h^2 \frac{\partial^3 h}{\partial x^3} \right]$$

$$+ \frac{2}{3} \xi K \left( 1 - \frac{1}{\epsilon_f} \right) \frac{\partial}{\partial x} \left( h^3 \frac{\partial}{\partial x} [E_{on}^v]^2 \right) = 0 \quad (5)$$

Upon linearization, one can perform an Orr-Sommerfeld stability analysis, leading for long waves to the critical Reynolds number:

$$Re < \frac{5}{6} \cot(\beta) - \frac{10K}{9} H^2 \frac{\left( 1 - \frac{1}{\epsilon_f} \right)^2}{\left( \frac{1}{\epsilon_f} + H - 1 \right)^3} \quad (6)$$

where H is the height of the electrode above the mean film surface. If K = 0, the Yih-Benjamin (Refs. 9,10) expression for the critical Reynolds number is obtained.

It is found that shocks can form, after which the lubrication equation is no longer valid. Nevertheless, with a finite length electrode these disturbances can wash harmlessly out from under the electrode and be carried away downstream. For large Reynolds numbers, Karman-Pohlhausen-type equations are derived:

$$\frac{\partial h}{\partial t} = - \frac{\partial q}{\partial x} \quad (7)$$

$$\frac{\partial q}{\partial t} + \frac{\partial}{\partial x} \left( \frac{6}{5} \frac{q^2}{h} \right) = \frac{2\bar{K}h}{R} \frac{\partial}{\partial x} (E_n^v)^2 - \frac{3}{R} \frac{q}{h^2} + \frac{\cos(\beta)}{FR^2} \left( \frac{h}{B} - h \frac{\partial h}{\partial x} \right). \quad (8)$$

where q is the volumetric flow rate per unit width.

From these equations a dispersion equation and steady-state solutions can be developed. Finally, numerical solutions, using the SOLA code, and a realistic electrostatic field, based on a single finite-length electrode rather than an infinite-length electrode, are obtained.

## RESULTS

The linear eigenvalue problem was solved numerically by a shooting method. Figure 2 shows the neutral stability curves (i.e.  $c_i = 0$ ) in the  $\alpha$  - Re plane for the case of  $Ca = \infty$ , i.e., no surface tension, and for  $Ca = 2 \times 10^{-4}$ , and for both K = 0, (no electric field) and K = 253.1. It is seen that small Reynolds number flows are stable, while increasing Re for fixed wavelength will cause the flow to become unstable. Note that the effect of the electric field is to lower the value of the critical Reynolds number at which the flow becomes unstable. Also note that the surface tension increases the critical Reynolds number for both K = 0 and K not equal to zero.

We plot the steady solution of both the lubrication model with  $Ca = \infty$  in Figure 3 and the steady-state Karman-Pohlhausen approximation in Figure 4. In order to simulate a slowly-varying potential we set  $\Phi =$

$\exp(-100x^2)$ . This potential has the slowly varying form of the assumptions for  $\xi$  small. The steady lubrication model is a second-order ordinary differential equation, which can be integrated once to reduce it to a first order equation. With the higher Reynolds numbers, the tendency to form a shock is greatly reduced and the wave shapes and amplitudes are in fair agreement, despite the small Reynolds number implicit in the lubrication model. Figure 5 shows the development of the surface wave at  $Re = 189$ , as determined by numerical solution of the full equations. It is seen that a peak height is reached with time at an amplitude of about 30% of the mean layer thickness. However, since the electrode is far from the film, the wave will be washed harmlessly downstream out of the influence of the electrode. Figure 6 shows the pressure at the bottom of the liquid layer directly over the puncture, where  $p = 0$  in the vacuum. The combined dimensionless vapor pressure and hydrostatic head tending to drive the liquid out of the hole is more than balanced by tension (negative pressure) due to the electric field with a margin of about two. The tension can be made larger by increasing the field strength and/or the Reynolds number. Figures 7 and 8 show that the Karman-Pohlhausen method gives quite similar results to the direct numerical solution.

### CONCLUSIONS

Calculations by three methods show that leaks from a liquid-film radiator with internal electrodes and surface puncture detection can be stopped with a safety factor of at least two. An experiment on earth is being undertaken to test some of these predictions and to determine feasible operating conditions.

### ACKNOWLEDGMENT

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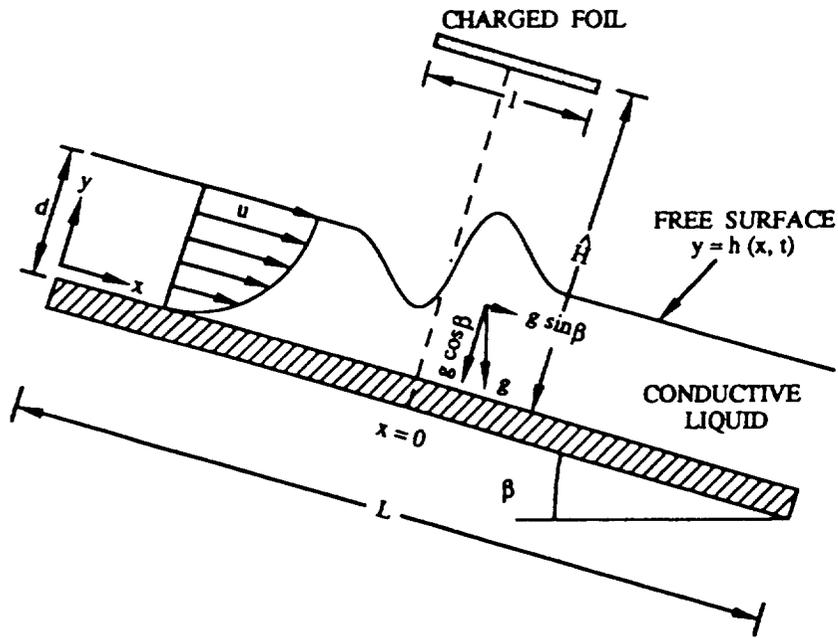


Fig. 1. The coordinate scheme of the plane flow with  $x = 0$  as puncture location.

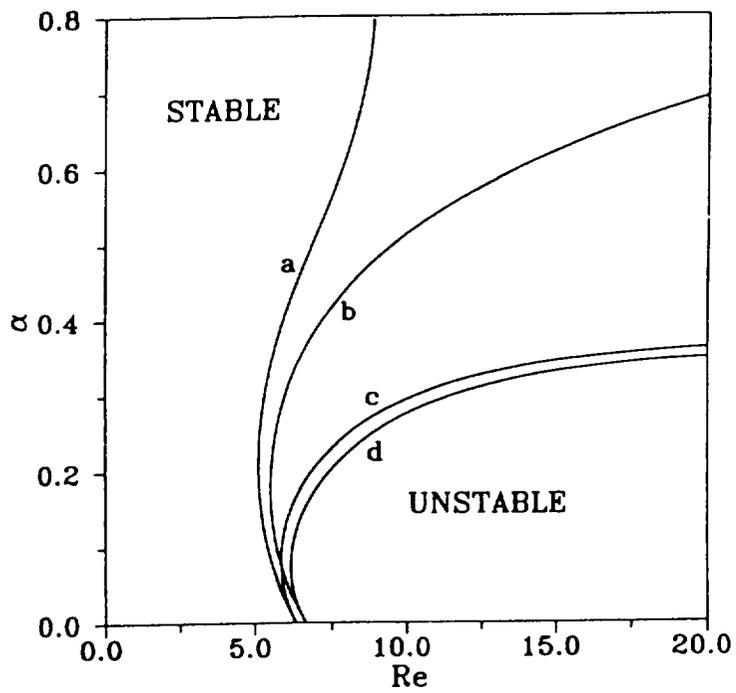


Fig. 2. Neutral stability curves in the  $\alpha - Re$  plane for  $\beta = 0.1$  radians,  $H = 13 \frac{1}{3}$  (a:  $K = 253.1$  and  $Ca = \infty$ , b:  $K = 0$  and  $Ca = \infty$ , c:  $K = 253.1$  and  $Ca = 2 \times 10^{-4}$  and d:  $K = 0$  and  $Ca = 2 \times 10^{-4}$ ).

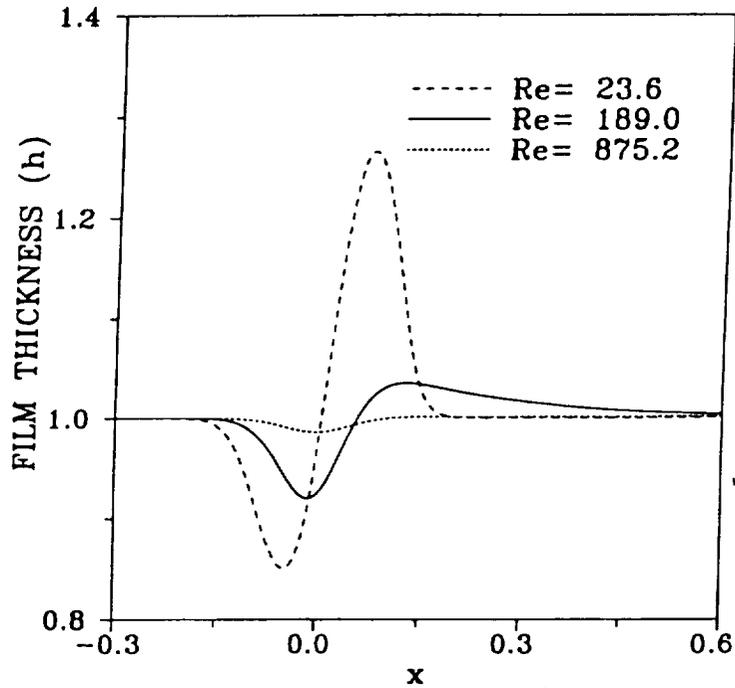


Fig. 3. Steady state solutions in the lubrication model with zero surface tension,  $F = 20\text{KV/cm}$  and  $\beta = 0.1$  rad ( $Re = 23.6, 189.0, 875.2$ ).

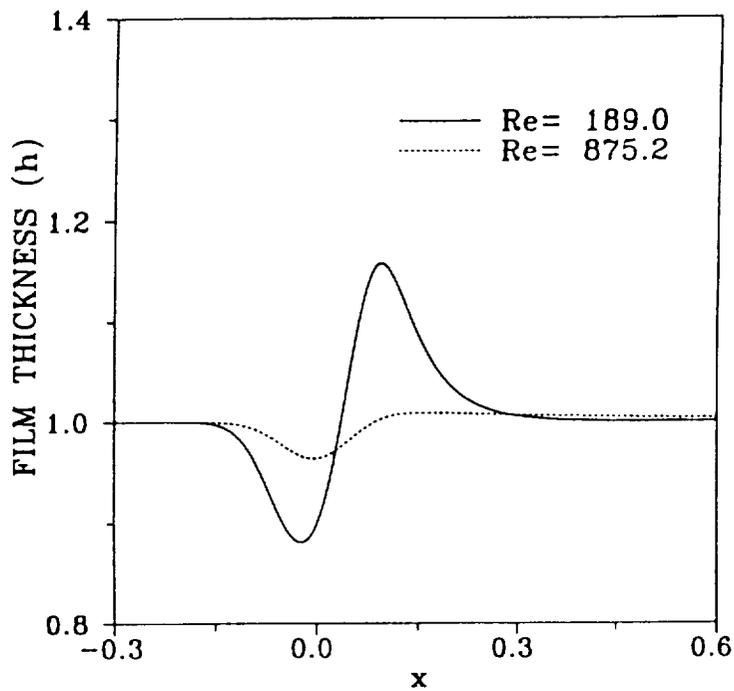


Fig. 4. Steady state solutions in the Karman-Pohlhausen model with zero surface tension,  $F = \text{KV/cm}$  and  $\beta = 0.1$  rad ( $Re = 189.0, 875.2$ ).

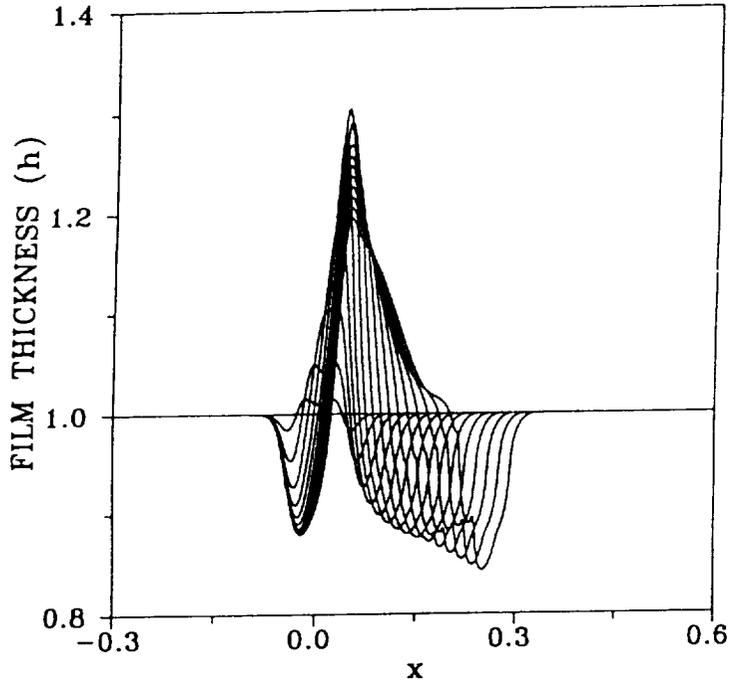


Fig. 5. Free surface  $h$  vs.  $x$  as determined by (2-5) - (2-13) for  $t = n(0.01)$ ,  $n = 1, \dots, 15$  with  $F = 20.0$  KV/cm,  $\beta = 0.1$  rad,  $d = 0.15$  cm,  $g = 100$  cm/s<sup>2</sup>,  $\sigma = 0$ ,  $Re = 189.0$ ,  $K = 28.87$ ,  $H = 13 \frac{1}{3}$  and the other parameter for lithium at 700°K.

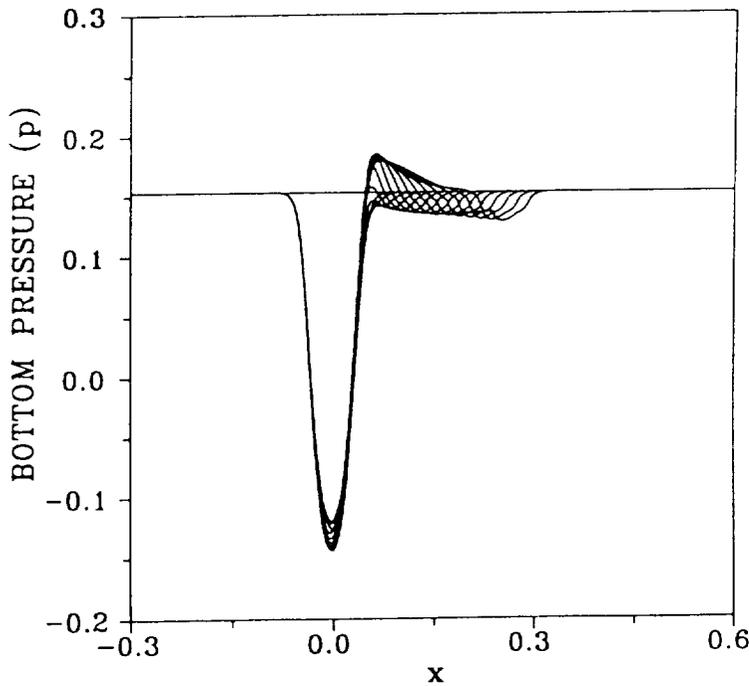


Fig. 6. Bottom pressure  $p$  vs.  $x$  as determined by (2.5) - (2-13) for  $t = n(0.01)$ ,  $n = 1, \dots, 15$  with  $F = 20.0$  KV/cm,  $\beta = 0.1$  rad,  $d = 0.15$  cm,  $g = 100$  cm/s<sup>2</sup>,  $\theta = 0$ ,  $Re = 189.0$ ,  $K = 28.87$ ,  $H = 13 \frac{1}{3}$  and the other parameter for lithium at 700° K.

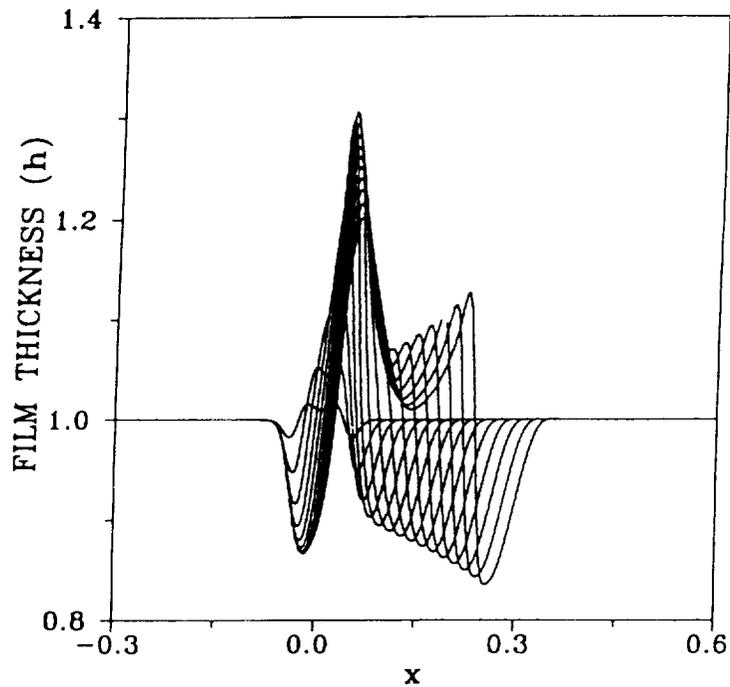


Fig. 7. Free surface  $h$  vs.  $x$  as determined by the Karman-Pohlhausen model (2.46) - (2-47) for  $t = n(0.01)$ ,  $n = 1, \dots, 15$  with  $F = 20.0$  KV/cm,  $\beta = 0.1$  rad,  $d = 0.15$  cm,  $g = 100$  cm/s<sup>2</sup>,  $\theta = 0$ ,  $Re = 189.0$ ,  $K = 28.87$ ,  $H = 13 \frac{1}{3}$  and the other parameter for lithium at  $700^\circ$  K

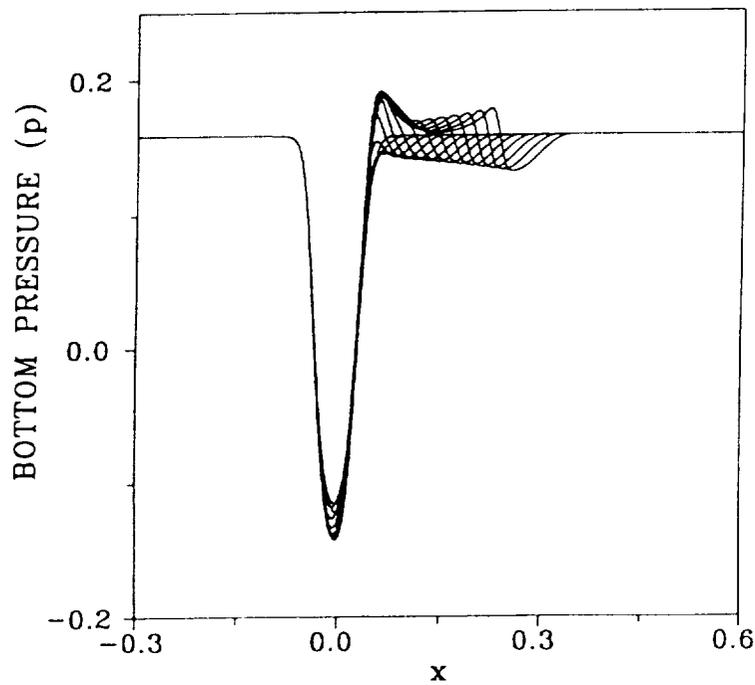


Fig. 8. Bottom pressure  $p$  vs.  $x$  as determined by the Karman-Pohlhausen model (2.46) - (2-47) for  $t = n(0.01)$ ,  $n = 1, \dots, 15$  with  $F = 20.0$  KV/cm,  $\beta = 0.1$  rad,  $d = 0.15$  cm,  $g = 100$  cm/s<sup>2</sup>,  $\theta = 0$ ,  $Re = 189.0$ ,  $K = 28.87$ ,  $H = 13 \frac{1}{3}$  and the other parameter for lithium at  $700^\circ$  K.